UCRL- 92417
PREPRINT

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This paper was prepared for submittal to
IEEE 24th Conference on
Decision and Control
Fort Lauderdale, FL
December 11-13, 1985

August 28, 1985

Lawrence Livermore National Laboratory

ADAPTIVE CONTROL OF DECENTRALIZED SYSTEMS: KNOWN SUBSYSTEMS, UNKNOWN INTERCONNECTIONS *

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ABSTRACT

One of the common assumptions in control of large interconnected systems is that models of subsystems are to a large extent known to a designer, and an essential modeling uncertainty resides in the interconnections. Current decentralized adaptive control schemes take no advantage of this fact. In this paper, an algorithm is presented in which adaptation of local feedback gains is in the a direction which compensates for the unknown interconnections, while exploiting the knowledge about the subsystems. As a result, we broaden considerably the class of interconnected systems for which decentralized adaptation is feasible.

* Work performed under the auspices of the U.S. Department of Energy by the Lawrence Livermore National Laboratory under contract number W-7405-ENG-48, and supported in part by the DOE Office of Basic Energy Sciences, Engineering Research Program, and the National Science Foundation Grant ECS-8315327.

I. INTRODUCTION

In modeling of large systems, it is standard to divide the system into a number of interconnected subsystems. A main advantage of this approach is that the modeling process is used to reflect our knowledge about the isolated parts of the system and that, as a consequence, the essential uncertainty resides in the interconnections. Control design is greatly simplified since the large problem has been broken into several smaller ones. Decentralized controllers have been shown to be robust to a wide range of nonlinear and time-varying elements in the subsystem interactions [1,2].

In spite of this progress in decentralized design, there still remain situations where fixed decentralized control laws may not provide the desired degree of robustness. For this reason, the adaptive control methods for single-input, single-output (SISO) systems [3] have been extended to decentralized systems [4-6]. These schemes are designed to stabilize an interconnection of uncertain or completely unknown subsystems. The ignorance of subsystem dynamics appears to be an artifice, however, which was used to make the centralized adaptive algorithms work in a decentralized setting, rather than to reflect a genuine modeling uncertainty about the subsystems. Furthermore, this artifice may unnecessarily prevent extensions of decentralized adaptive control to the broader class of interconnected multi-input, multi-output (MIMO) subsystems.

The purpose of this paper is to explore the possibility of adaptively controlling *known* MIMO subsystems so as to stabilize the overall system, or track a reference model, in spite of the uncertain interconnections between the subsystems. The adaptive schemes presented here, like those in [4-6], are based on model-reference adaptive control. In this approach, parameters of the controller are varied to try to match each closed-loop subsystem to a reference model's behavior. We note that in the decentralized model reference scheme, the overall model, consisting of isolated submodels, differs in *structure* from the interconnected large-scale system, hence no variation of parameters will allow exact model matching. Instead we rely on high gain feedback to cancel the effect of interconnection disturbances and force the system to closely follow the reference signals.

II. PROBLEM STATEMENT

We consider the problem of controlling N interconnected subsystems

$$S_i: \quad \dot{x}_i = A_i x_i + B_i u_i + \sum_{j=1}^{N} A_{ij} x_j \quad , \quad i \in \mathcal{N} \tag{2.1}$$

where $x_i(t) \in \mathcal{R}^{n_i}$ is the state and $u_i(t) \in \mathcal{R}^{p_i}$ is the input to S_i at time $t \in \mathcal{R}$, and $\mathcal{N} = \{1, 2 \ldots, N\}$. A_i, B_i, and A_{ij} are constant matrices of appropriate dimension. The overall system of equations (2.1) can be represented in block matrix form as

$$S: \quad \dot{x} = A_D x + A_C x + Bu \quad , \tag{2.2}$$

where $x = (x_1^T, x_2^T, \ldots, x_N^T)^T$, $u = (u_1^T, u_2^T, \ldots, u_N^T)^T$, $A_D = \text{diag}\{A_1, A_2, \ldots, A_N\}$, $B = \text{diag}\{B_1, B_2, \ldots, B_N\}$, and $A_C = [A_{ij}]$.

We shall assume that the systems S is partially unknown, and thus some form of adaptive control is required. Furthermore, we assume that all of the modeling uncertainty is in the interconnection matrix, A_C, but that the subsystem matrices A_i and B_i are known.

The control laws are restricted to be decentralized, that is each controller operates on its local subsystem exclusively, with no exchange of information between subsystems. In the adaptive controllers to be analyzed in this paper, this means that not only the feedback control law, but also the adaptation mechanism, must involve local information only.

As is well known, even in the non-adaptive case, decentralized stabilization of S is not in general possible with an arbitrary A_C. We shall assume a certain structure to A_C which makes stabilization possible. In the first algorithm presented, we require that the A_{ij} be factorable as

$$A_{ij} = B_i H_{ij} \quad , \quad i, j \in \mathcal{N} \tag{2.3}$$

where $H_{ij} \in \mathcal{R}^{p_i \times n_j}$ are bounded but otherwise arbitrary matrices. This is equivalent to requiring that:

$$\mathcal{R}(A_{ij}) \in \mathcal{R}(B_i) \quad , \quad i, j \in \mathcal{N} \tag{2.4}$$

where $\mathcal{R}(\cdot)$ denotes the range space of the indicated matrix.

We will be applying the direct adaptive control philosophy in the schemes presented in this paper. In the direct approach, the feedback gain vector is adapted directly, as opposed to a two step process of system identification followed by control design. Direct adaptive control has been more successful than the indirect method for large-scale systems because the difficult intermediate step of subsystem identification is avoided. In the direct method, a reference model must be specified:

$$\dot{\bar{x}}_i = \bar{A}_i \bar{x}_i + \bar{B}_i r_i \quad , \quad i \in \mathcal{N} \tag{2.5}$$

where $\bar{x}_i(t) \in \mathcal{R}^{n_i}$, and $r_i(t) \in \mathcal{R}^{p_i}$ is a bounded piecewise continuous reference signal. Equivalently:

$$\dot{\bar{x}} = \bar{A}_D \bar{x} + \bar{B} r \tag{2.6}$$

where $\bar{x} = \left(\bar{x}_1^T, \bar{x}_2^T, \ldots, \bar{x}_N^T\right)^T$, $r = \left(r_1^T, r_2^T, \ldots, r_N^T\right)^T$, $\bar{A}_D = \text{diag}\{\bar{A}_i\}$, $\bar{B} = \text{diag}\{\bar{B}_i\}$.

The matrices $\bar{A}_i$ and $\bar{B}_i$ are computed as follows. Since we know the subsystem plant models, we can require that $\bar{B}_i = B_i$, $i \in \mathcal{N}$. We then must provide a feedback matrix $\bar{K}_i \in \mathcal{R}^{p_i \times n_i}$ which satisfies

$$\bar{A}_i = A_i - B_i \bar{K}_i \quad , \quad i \in \mathcal{N} \quad . \tag{2.7}$$

so that the $\bar{A}_i$, $i \in \mathcal{N}$, are stable. These $\bar{K}_i$ will be known to the designer. The condition (2.7) is called the model matching condition, and $\bar{K}_i$ the model matching gains. It is well known that choice of $\bar{K}_i$ can be used to produce arbitrarily stable $\bar{A}_i$.

The state feedback law implied by (2.7):

$$u_i = -\bar{K}_i x_i + r_i \quad , \tag{2.8}$$

stabilizes the isolated subsystems, although it may not stabilize the system as a whole due to the effect of interconnecting signals, $A_{ij} x_j$. Note that this is quite a different situation than in ordinary model reference adaptive control, where it is assumed that there exist feedback gains that will exactly match the plant to a stable reference model. Decentralized adaptive controllers instead must rely on increasing the stability of the local subsystems (through, possibly, high gains) in order to stabilize to the overall system.

III. ADAPTIVE SCHEME

The adaptive scheme to be presented is based on increasing optimal feedback gains until overall stability is achieved. The optimal* feedback gain for subsystem i is:

$$K_i' = R_i^{-1} B_i^T P_i \quad , \tag{3.1}$$

where $R_i \in \mathcal{R}^{p_i \times p_i}$, $R_i = R_i^T > 0$, and P_i is the positive definite solution to the algebraic Liapunov equation:

$$P_i \bar{A}_i + \bar{A}_i^T P_i = -Q_i \quad , \tag{3.2}$$

where $Q_i = Q_i^T > 0$. The control to be used for subsystem i is:

$$u_i = -\bar{K}_i x_i - \alpha_i K_i' e_i + r_i \quad , \tag{3.3}$$

where $e_i = x_i - \bar{x}_i$. Equivalently:

$$u = -\bar{K} x - \alpha K' e + r \quad , \tag{3.4}$$

where $e = \left(e_1^T, e_2^T, \ldots, e_N^T\right)^T$ $\alpha = \mathrm{diag}\{\alpha_i I_{p_i}\}$, $\bar{K} = \mathrm{diag}\{\bar{K}_1, \bar{K}_2, \ldots, \bar{K}_N\}$, $K' = \mathrm{diag}\{K_1', K_2', \ldots, K_N'\}$, and I_{p_i} represents the $p_i \times p_i$ identity matrix. The α_i are parameters to be locally adapted. Combining the control (3.4) with the plant (2.2), we get:

$$\dot{x} = (A_D - BK)x - \alpha B K' e + Br \quad . \tag{3.5}$$

Using (2.6) an error differential equation can be written

$$\dot{e} = \bar{A}_D e - \alpha B K' e + A_C e + A_C \bar{x} \quad . \tag{3.6}$$

For preliminary analysis, consider the unforced case, $r_i(t) = 0$ for all t. This implies $\bar{x}_i(t) = 0$.

Theorem 3.1

If we use adaptation laws

$$\dot{\alpha}_i = \gamma_i^{-1} e_i^T P_i B_i R_i^{-1} B_i^T P_i e_i \quad , \quad \alpha_i(0) \geq 0 \tag{3.7}$$

where $\gamma_i > 0$, then $e(t) \to 0$ as $t \to \infty$ and the $\alpha_i(t)$ are uniformly bounded.

Proof. Use the candidate vector Liapunov function

$$v_i(e_i, \alpha_i) = e_i^T P_i e_i + \gamma_i \tilde{\alpha}_i^2 \quad . \tag{3.8}$$

The symbol $\tilde{\alpha}_i$ represents the difference between $\alpha_i(t)$ and a "sufficiently stabilizing" constant α_i^*. Taking the time derivative:

$$\begin{aligned} \dot{v}_i = & - e_i^T Q_i e_i - 2\alpha_i e_i^T P_i B_i K_i' e_i \\ & + 2 e_i^T P_i B_i \sum_{j=1}^{N} H_{ij} e_j \quad , \\ & + 2\tilde{\alpha}_i e_i^T P_i B_i R_i^{-1} B_i^T P_i e_i \end{aligned} \tag{3.9}$$

* K_i' is the optimal state feedback gain for the isolated, model-matched, subsystem only. In the interconnected case, K_i' is a suboptimal decentralized feedback gain.

using $K_i' = R_i^{-1} B_i^T P_i$ and $\tilde{\alpha}_i = \alpha_i - \alpha_i^\star$, and completing the square:

$$\begin{aligned} \dot{v}_i \leq & - e_i^T \left(Q_i + \alpha_i^\star P_i B_i R_i^{-1} B_i^T P_i \right) e_i \\ & + \alpha_i^{\star^{-1}} \left\| \sum_{j=1}^{N} R_i^{1/2} H_{ij} e_j \right\|^2 \end{aligned} \quad . \tag{3.10}$$

Let

$$V(e, \alpha) = \sum_{i=1}^{N} v_i \quad , \tag{3.11}$$

and compute

$$\begin{aligned} \dot{V} & \leq -e^T Q e + e^T H^T R^{1/2} \alpha^{\star^{-1}} R^{1/2} H e \quad , \\ & \leq \left[-\lambda_m (Q) + \lambda_M \left(H^T R H \right) \max_i \{ \alpha_i^{\star^{-1}} \} \right] \|e\|^2 \end{aligned} \quad , \tag{3.12}$$

where $\lambda_m(\cdot)$ and $\lambda_M(\cdot)$ represent the minimum and maximum eigenvalues of the indicated matrix, respectively. Therefore, $\dot{V} \leq 0$ if $\alpha^\star$ is chosen sufficiently large. This implies (e, α) goes to the invariant set, $\|e\| = 0$, and both $e(t)$ and $\alpha(t)$ are uniformly bounded. This completes the proof.

The proposed adaptive scheme has the advantage that it is very simple, involving only one adaptive parameter per subsystem. Also, the subsystems are allowed to have multiple control inputs, a relaxation of a constraint to single input systems imposed by earlier decentralized adaptive controllers.

IV. TRACKING

In the case where model tracking is required, $\bar{x}(t)$ is not identically zero and therefore acts as a disturbance term through the interconnections to drive the error model (3.6).

$$\dot{e} = \bar{A}_D e - \alpha B K' e + A_C e + A_C \bar{x} \quad . \tag{3.6}$$

This disturbance prevents exact model following but we can make the error bounded. We modify the adaptation law slightly and obtain the following theorem.

Theorem 4.1

If we use the adaptation laws

$$\dot{\alpha}_i = \gamma_i^{-1} \left[e_i^T P_i B_i R_i^{-1} B_i^T P_i e_i - \sigma_i \alpha_i \right] \quad ; \quad \alpha_i(0) \geq 0 \quad , \tag{4.1}$$

where $\sigma_i > 0$, then the solution $(e(t), \alpha(t))$ is globally ultimately bounded.

Proof. Let $v_i(e_i, \alpha_i) = e_i P_i e_i + \gamma_i \tilde{\alpha}_i^2$. Along (3.6) and (4.1):

$$\begin{aligned} \dot{v}_i \leq & - e_i^T Q_i e_i \\ & + 2 e_i^T P_i B_i H_i e_i - \alpha_i^\star e_i^T P_i B_i R_i^{-1} B_i^T P_i e_i \\ & + 2 e_i^T P_i B_i H_i \bar{x}_i - \alpha_i^\star e_i^T P_i B_i R_i^{-1} B_i^T P_i e_i \\ & - 2 \sigma_i \tilde{\alpha}_i \alpha_i^\star - \sigma_i \tilde{\alpha}_i^2 \end{aligned} \quad . \tag{4.2}$$

Completing squares

$$\dot{v}_i \leq - e_i^T Q_i e_i + {\alpha_i^\star}^{-1} e_i^T H_i^T R_i H_i e_i - \sigma_i \tilde{\alpha}_i^2 \\ + {\alpha_i^\star}^{-1} \bar{x}_i^T H_i^T R_i H_i \bar{x}_i + \sigma_i {\alpha_i^\star}^2 \tag{4.3}$$

Choosing $V = \sum_{i=1}^N v_i$ we find that $\dot{V} \leq 0$ if $(e(t), \alpha(t))$ lies outside a bounded region. This implies $e(t)$ and $\alpha(t)$ are globally ultimately bounded and proves the theorem.

Corollary 4.1

Tracking error, $e(t)$, converges to a residual set, $\Omega_e = \{e \in \mathcal{R}^n : \ \|e\| < \xi\}$, and ξ can be made as small as desired through choice of σ_i.

Proof. From (4.3) we can see that

$$\xi < \lambda_m^{-1/2}(Q) \sup_t \|\bar{x}\| \ \lambda_M(H^T R H)(\underline{\alpha}^\star)^{-1} + \bar{\sigma}{\bar{\alpha}^\star}^2 \quad , \tag{4.4}$$

where $\underline{\alpha}^\star = \min_i(\alpha_i^\star)$, $\bar{\alpha}_i^\star = \max_i(\alpha_i^\star)$, $\bar{\sigma} = \max_i(\sigma_i)$. We choose $\underline{\alpha}^\star$ large enough to make the first term small, and $\bar{\sigma}$ small enough to make the second term small, so (4.4) is satisfied for any given $\xi > 0$. Note that the small $\bar{\sigma}$ will lead to possibly large values for $\alpha(t)$ which is intuitively reasonable since we would expect tighter tracking to require higher feedback gains.

V. OUTPUT FEEDBACK

If only the outputs of subsystems are available for feedback then some form of dynamic output feedback compensator is required. Under certain conditions, the decentralized observer presented in [7] can be used. These conditions are summarized below.

The measured subsystem outputs are given by:

$$y_i = C_i x_i \quad , \tag{5.1}$$

where $y_i \in \mathcal{R}^{m_i}$. We construct a dynamic compensator of the form

$$\dot{z}_i = F_i z_i + G_i y_i - \bar{G}_i u_i \quad , \tag{5.2}$$

$$u_i = (\alpha_i + 1)(Y_i y_i + Z_i z_i) \quad , \tag{5.3}$$

where $z_i \in \mathcal{R}^{r_i}$, $r_i \leq n_i$, and F_i, $i \in \mathcal{N}$ are stable matrices.

For ease in presenting the argument below, let us choose the reference model of (2.5) so that $\bar{K}_i = K_i' = R_i^{-1} B_i^T P_i$ (this gain is known to stabilize the isolated subsystem). Therefore the gains would be $K_i = (\alpha_i + 1) K_i'$ if the local state variables were available for feedback.

In order for output feedback eventually to have the same effect as state feedback we require that

$$u_i(t) \rightarrow K_i x_i(t) \qquad \text{as } t \rightarrow \infty \quad , \tag{5.4}$$

which can be satisfied if

$$z_i(t) \rightarrow T_i x_i(t) \quad , \tag{5.5}$$

and we choose Y_i and Z_i so that

$$[Y_i \ Z_i] \begin{bmatrix} C_i \\ T_i \end{bmatrix} = K_i' \quad . \tag{5.6}$$

The estimator will converge as in (5.5), subject to the observer dynamics (5.2), system dynamics (2.1), and interconnection structure (2.3) if and only if F_i, T_i, G_i, and $\bar{G}_i$ be selected so that

$$F_i T_i - T_i A_i + G_i C_i = 0 \quad , \tag{5.7}$$

$$T_i B_i = 0 \quad , \tag{5.8}$$

$$\bar{G}_i = 0 \quad , \tag{5.9}$$

as was shown in [7]. Furthermore, Y_i and Z_i must exist to satisfy (5.6). That is, it is necessary and sufficient that

$$\mathcal{N}\left(K_i'\right) \supset \mathcal{N}\left(\begin{bmatrix} C_i \\ T_i \end{bmatrix}\right) = \mathcal{N}(T_i) \cap \mathcal{N}(C_i) \quad , \tag{5.10}$$

where $\mathcal{N}(\cdot)$ denotes the null space of the indicated matrix. Constructive methods for computing F_i, T_i, G_i, Y_i, and Z_i (if they exist) are given in [7].

When these conditions can be satisfied, output feedback through the dynamic compensators, (5.2), (5.3), is assymptotically equivalent to state feedback, and the control law $u_i = (\alpha + 1)\left(Y_i y_i + Z_i z_i\right)$ can be used with the adaptive schemes presented in this paper.

VI. CONCLUSION

We have provided adaptive control schemes for decentralized large-scale systems that exploit *a-priori* knowledge about the subsystems and adapt to compensate for uncertainty in the interconnections. Such schemes utilize the high gain feedback approach which requires that interconnection structure to be restricted to a certain class, that is, interconnecting signals must be in the range space of the control input.

The adaptive decentralized regulator in this context has been proven globally asymptotically stable, and the model tracking algorithm can be made to follow a reference signal to any degree of accuracy required.

Improvements over earlier schemes include a simplified adaptation algorithm and the ability to handle MIMO subsystems, using decentralized state estimators for output feedback if necessary.

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